c) $\forall n \in \mathbb{N} \quad V_n = V_0 \times q^n \qquad -1 < \frac{1}{3} < 1$

or $V_0 = U_1 - \frac{1}{3} U_0$ et $q = \frac{1}{3}$

Donc $\boxed{V_n = \left(U_1 - \frac{1}{3} U_0\right) \cdot \left(\frac{1}{3}\right)^n \quad \forall n \in \mathbb{N}}$

$\lim\limits_{n \to +\infty} V_n = \lim\limits_{n \to +\infty} \left(U_1 - \frac{1}{3} U_0\right) \cdot \left(\frac{1}{3}\right)^n = 0$

$\left(\frac{1}{3}\right)^n \to 0$ car $-1 < \frac{1}{3} < 1$

D'où $\boxed{\lim\limits_{n \to +\infty} V_n = 0}$

2°) a) $\forall n \geq 2,$ $\quad n - 1 - 1 + 1$

$$\sum_{k=1}^{n-1} V_k = V_1 \cdot \frac{1 - q^{n-1}}{1 - q}$$

$$= \frac{2}{9} \cdot \frac{1 - \left(\frac{1}{3}\right)^{n-1}}{1 - \frac{1}{3}}$$

$$= \frac{2}{9} \cdot \frac{3}{2} \left(1 - \left(\frac{1}{3}\right)^{n-1}\right)$$

$$= \boxed{\frac{1}{3} \cdot \left(1 - \frac{1}{3^{n-1}}\right)}$$